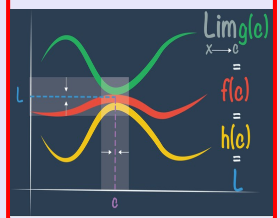
Calculus I

Lecture 48



Feb 19-8:47 AM

Formulas from Pre Calc or College Algebra

$$\boxed{\sum_{i=1}^{n} c = cn}$$

$$\boxed{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}}$$

$$\boxed{\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}}$$

$$\boxed{\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2}$$

$$\sum_{i=1}^{n}(a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$$

May 8-9:52 AM

Suppose $f(x) \geq 0$ and Continuous for all values from a to b.

The area below $f(x)$, above x-axis from $x=a$ to $x=b$ is given by

$$\boxed{A = \lim_{n \to \infty} \sum_{i=1}^{n} \Delta x \cdot f(x_i)}$$

where $\Delta x = \dfrac{b-a}{n}$, $x_i = a + i\Delta x$

May 9-8:50 AM

Find the area below $\underbrace{f(x) = 4 - x^2}$, and above x-axis

Parabola opens downward

Polynomial $\Rightarrow$ Cont. everywhere

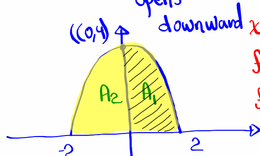
Y-Int $\to (0,4)$

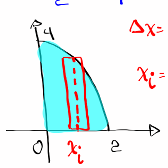
x-Int $\to (\pm 2, 0)$

$f(-x) = 4-(-x)^2 = 4 - x^2 = f(x)$

$f(x) \to$ even function

$\to$ Sym. w/t Y-axis





$\Delta x = \dfrac{b-a}{n} = \dfrac{2-0}{n} = \dfrac{2}{n}$

$x_i = a + i\Delta x = 0 + i \cdot \dfrac{2}{n} \Rightarrow x_i = \dfrac{2i}{n}$

$$A = 2 \lim_{n \to \infty} \sum_{i=1}^{n} \Delta x \cdot f(x_i)$$

$$A = 2 \lim_{n \to \infty} \sum_{i=1}^{n} \frac{2}{n}\left[4 - x_i^2\right] = 2 \lim_{n \to \infty} \sum_{i=1}^{n} \frac{2}{n}\left[4 - \left(\frac{2i}{n}\right)^2\right]$$

$$= 2 \cdot \lim_{n \to \infty} \frac{2}{n}\left(\sum_{i=1}^{n} 4 - \sum_{i=1}^{n} \frac{4i^2}{n^2}\right)$$

$$= 2 \lim_{n \to \infty} \frac{2}{n}\left[4n - \frac{4}{n^2} \cdot \frac{n(n+1)(2n+1)}{6}\right]$$

$$= 2 \lim_{n \to \infty}\left[8 - \frac{8 \cdot 2n^3 + \cdots}{6n^3}\right] = 2\left[8 - \frac{16}{6}\right] = 2\left[8 - \frac{8}{3}\right]$$

$$= 2\left[\frac{24}{3} - \frac{8}{3}\right] = 2 \cdot \frac{16}{3} = \boxed{\frac{32}{3}}$$

May 13-8:47 AM

Indefinite Integrals:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad , \; n \neq -1$$

$$\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$$

$$\int [4 - x^2] dx = \int 4 dx - \int x^2 dx$$

$$= 4x - \frac{x^3}{3} + C$$

Let's evaluate this answer at $\boxed{x=2}$ & $\boxed{x=-2}$ and find the difference.

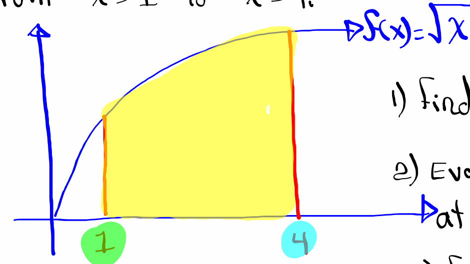
$$\left[4 \cdot 2 - \frac{2^3}{3} + C\right] - \left[4 \cdot (-2) - \frac{(-2)^3}{3} + C\right]$$

$$= 8 - \frac{8}{3} \cancel{+C} + 8 - \frac{8}{3} \cancel{-C} = 16 - \frac{16}{3} = \frac{48}{3} - \frac{16}{3}$$

$$= \boxed{\frac{32}{3}}$$

May 13-9:03 AM

Find the area below $f(x) = \sqrt{x}$, above x-axis from $x=1$ to $x=4$.



1) Find $\int f(x) dx$

2) Evaluate your answer at right point & Left point.

3) Find the difference.

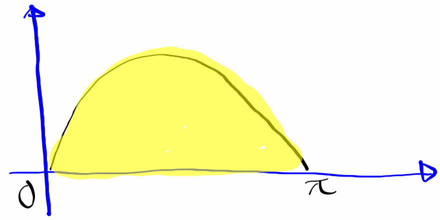
$$\int \sqrt{x}\, dx = \int x^{1/2} dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{x^{3/2}}{3/2} + C = \boxed{\frac{2}{3} x\sqrt{x} + C}$$

$$A = \left[\frac{2}{3} \cdot 4\sqrt{4} + C\right] - \left[\frac{2}{3} \cdot 1\sqrt{1} + C\right]$$

$$= \frac{2}{3} \cdot 4 \cdot 2 \cancel{+C} - \frac{2}{3} \cdot 1 \cdot 1 \cancel{-C} = \frac{2}{3}[8-1] = \boxed{\frac{14}{3}}$$

May 13-9:11 AM

Find the area below $f(x) = \sin x$, and above x-axis from $x=0$ to $x=\pi$.



1) $\int \sin x \, dx$

2) Eval. at $x=\pi$ ξ $x=0$

3) Find the difference.

$$\int \sin x \, dx = -\cos x + C$$

$$\left[-\cos\pi + C\right] - \left[-\cos 0 + C\right]$$

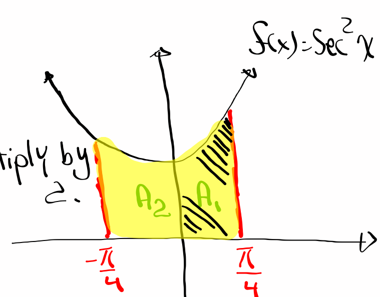
$$= 1 + \cancel{C} + 1 - \cancel{C} = \boxed{2}$$

May 13-9:19 AM

Find the area below $f(x) = \sec^2 x$, above x-axis from $x = \frac{-\pi}{4}$ to $x = \frac{\pi}{4}$.

Hint: $f(x)$ is an even function

$f(x) \geq 0$



Let's find $area_1$, then multiply by 2.

$$\int \sec^2 x \, dx = \tan x + C$$

Eval. at $x = \frac{\pi}{4}$ ξ $x=0$, then find difference.

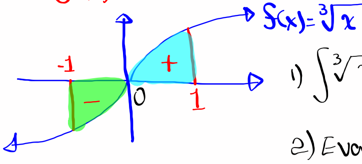
$$\left[\tan\frac{\pi}{4} + C\right] - \left[\tan 0 + C\right] = 1 + C - C = 1 \leftarrow A_1$$

Final Ans. $2A_1 = 2 \cdot 1 = \boxed{2}$

May 13-9:24 AM

Find the area bounded by $f(x)=\sqrt[3]{x}$ and x-axis from $x=-1$ to $x=1$.

Hint: $f(x)$ is an odd function $\Rightarrow$ Sym. w/t origin.



1) $\int \sqrt[3]{x}\,dx$

2) Eval. at $x=1$ & $x=0$

3) Find the diff.

4) Multiply by 2.

$$\int \sqrt[3]{x}\,dx = \int x^{1/3}\,dx$$

$$= \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C$$

$$= \frac{x^{4/3}}{4/3} + C = \frac{3}{4}x^{4/3} + C = \frac{3}{4}x\sqrt[3]{x} + C$$

Final Ans.

$$2\left[\left(\frac{3}{4}\cdot\cancel{1\cdot\sqrt[3]{1}}^{\,1}\ \cancel{+C}\right) - \left(\cancel{\frac{3}{4}\cdot 0\cdot\sqrt[3]{0}}^{\,0}\ \cancel{+C}\right)\right] = 2\cdot\frac{3}{4}$$

$$= \boxed{\frac{3}{2}}$$

May 13-9:31 AM

Definite integral:

Assuming $f'(x)$ is Cont. on $[a,b]$

$$\int_a^b f'(x)\,dx = f(x)\Big|_a^b = f(b) - f(a)$$

ex: $$\int_1^3 (x^2 - x)\,dx = \left(\frac{x^3}{3} - \frac{x^2}{2}\right)\Big|_1^3$$

$$= \left(\frac{3^3}{3} - \frac{3^2}{2}\right) - \left(\frac{1^3}{3} - \frac{1^2}{2}\right)$$

$$= \boxed{\frac{27}{3}} - \frac{9}{2} - \boxed{\frac{1}{3}} + \frac{1}{2}$$

$$= \frac{26}{3} - \frac{8}{2} = \frac{26\cdot 2 - 8\cdot 3}{6}$$

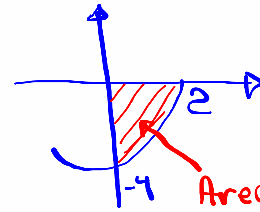
$$= \frac{52 - 24}{6} = \frac{28}{6}$$

$$= \boxed{\frac{14}{3}}$$

May 13-9:41 AM

Evaluate $\int_0^2 (x-2)(x+2)\,dx$

$$= \int_0^2 (x^2-4)\,dx = \left(\frac{x^3}{3} - 4x\right)\Bigg|_0^2$$



Area below x-axis

Can definite integral be negative? Yes

$$= \frac{2^3}{3} - 4(2) - \left(\frac{0^3}{3} - 4(0)\right)$$

$$= \frac{8}{3} - 8 - 0 = \frac{8}{3} - 8$$

$$= \frac{8}{3} - \frac{24}{3} = \boxed{-\frac{16}{3}}$$

If you are using definite integral to find area, then ans. is NO.

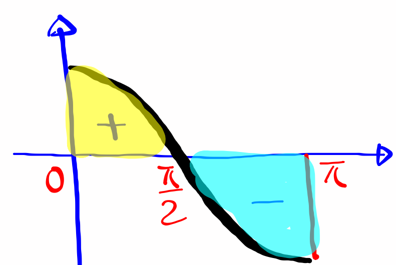
May 13-9:47 AM

Evaluate $\int_0^{\pi} \cos x\,dx$

$$= \sin x \Big|_0^{\pi}$$

$$= \sin\pi - \sin 0$$

$$= 0 - 0 = \boxed{0}$$



In terms of Area

$$2\int_0^{\pi/2} \cos x\,dx$$

May 13-9:53 AM